

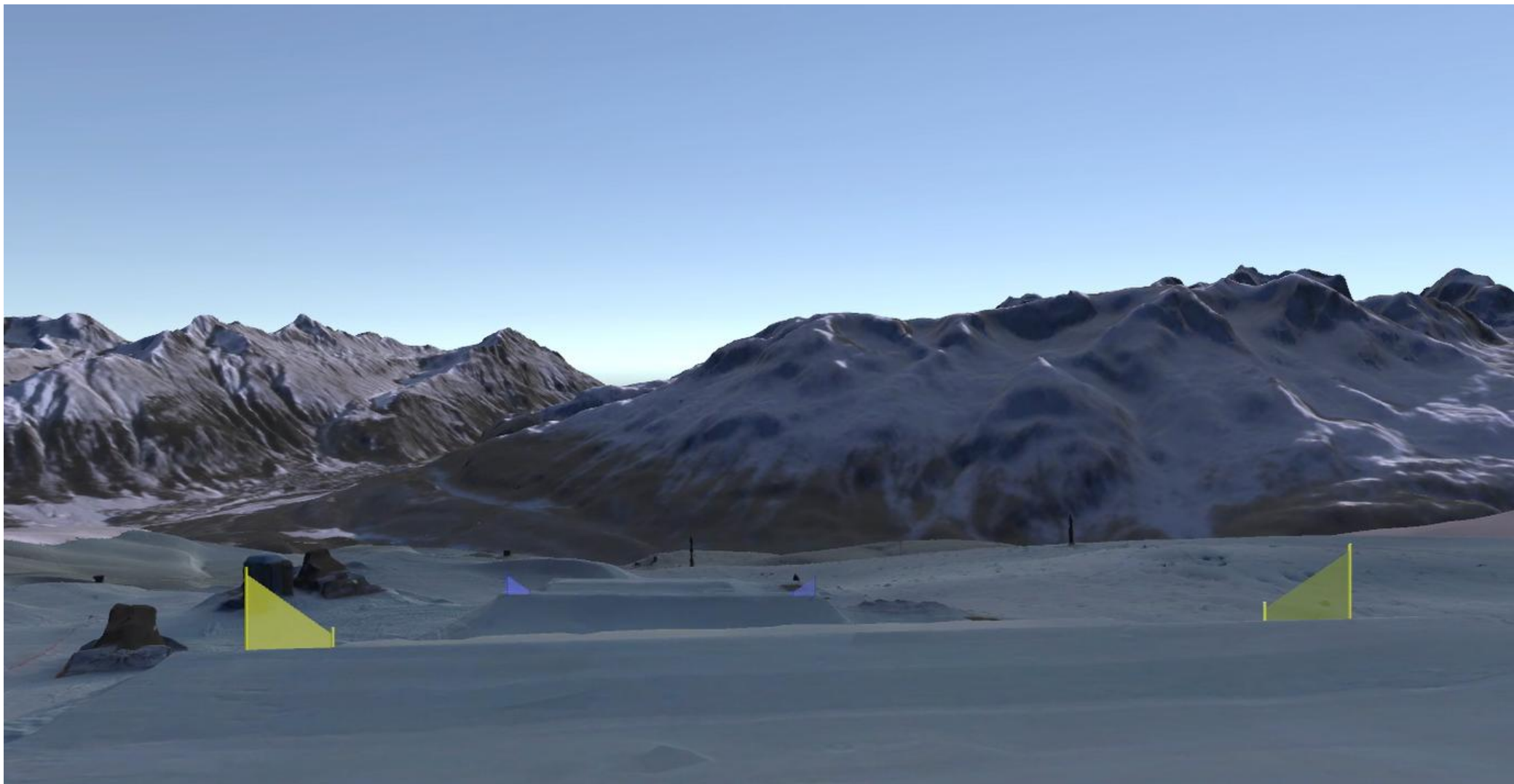
# TRAJECTORY FILTERING AND JUMP MODELLING IN SKI CROSS VR-APPLICATION

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<sup>2</sup> Swiss-Ski, Worblaufen, Switzerland

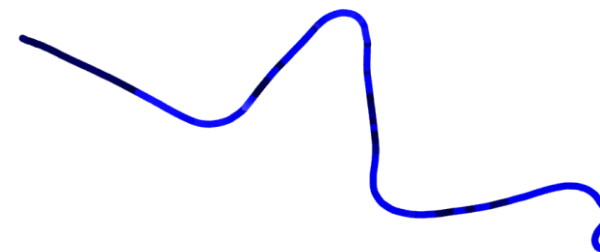
# Main goal : First person VR-App for ski cross



# What we need



3D-model (high precision : ~0.1m)

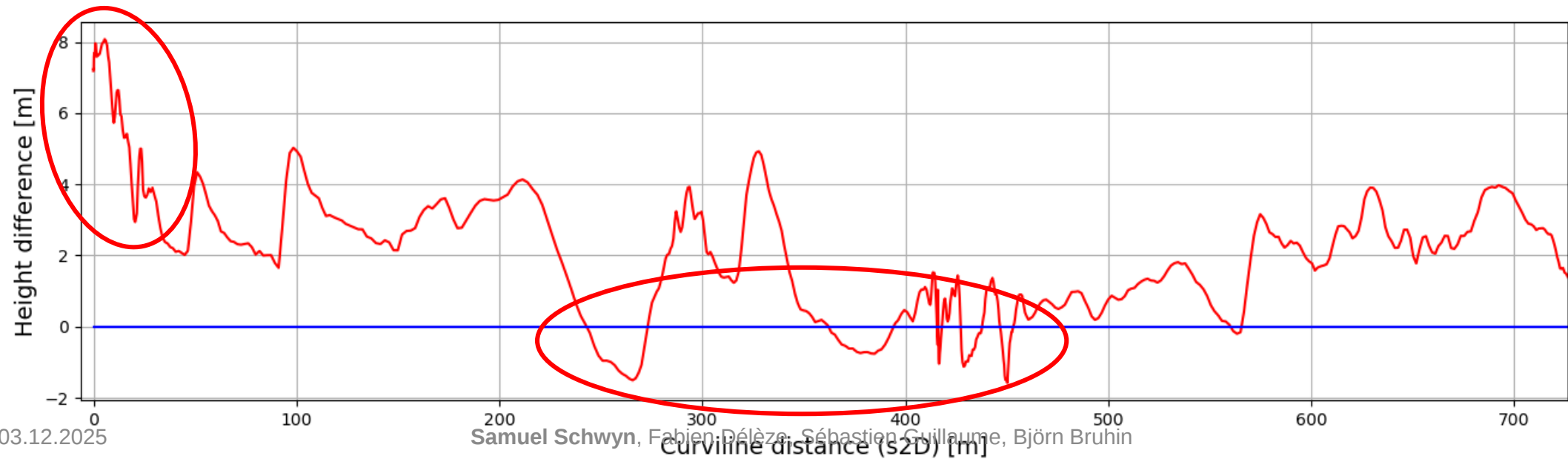
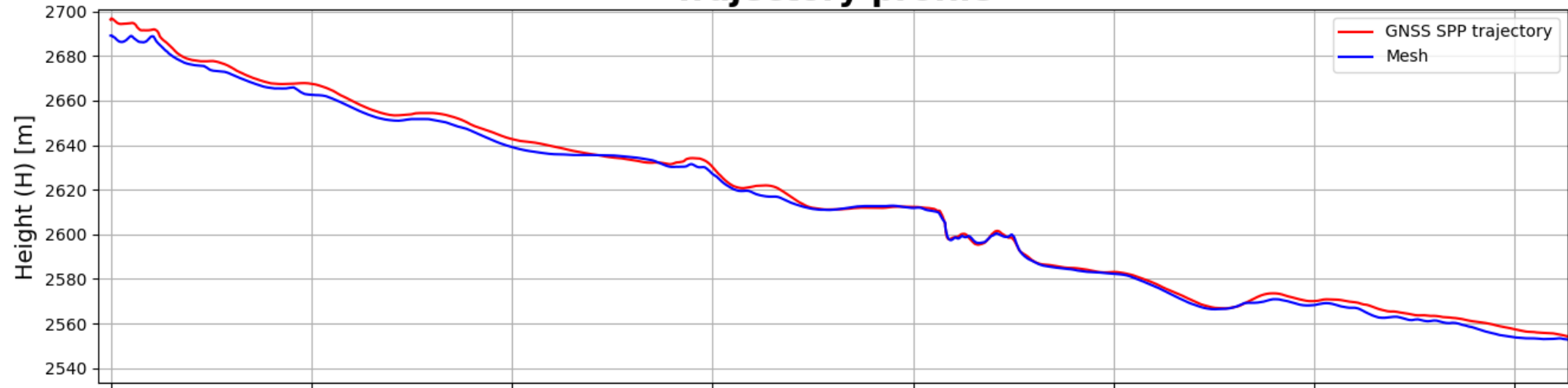


$$\mathbf{x}(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}$$

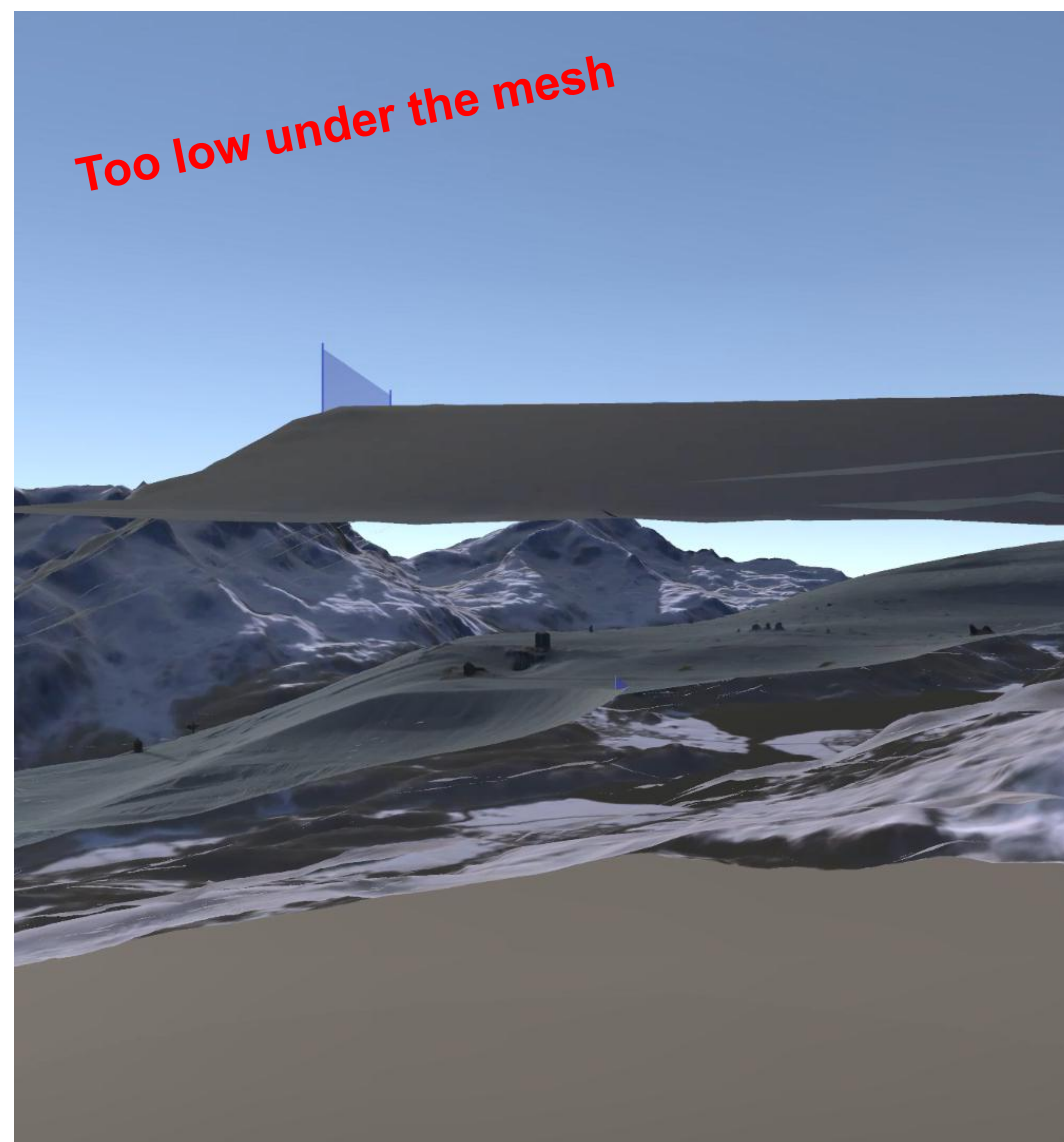
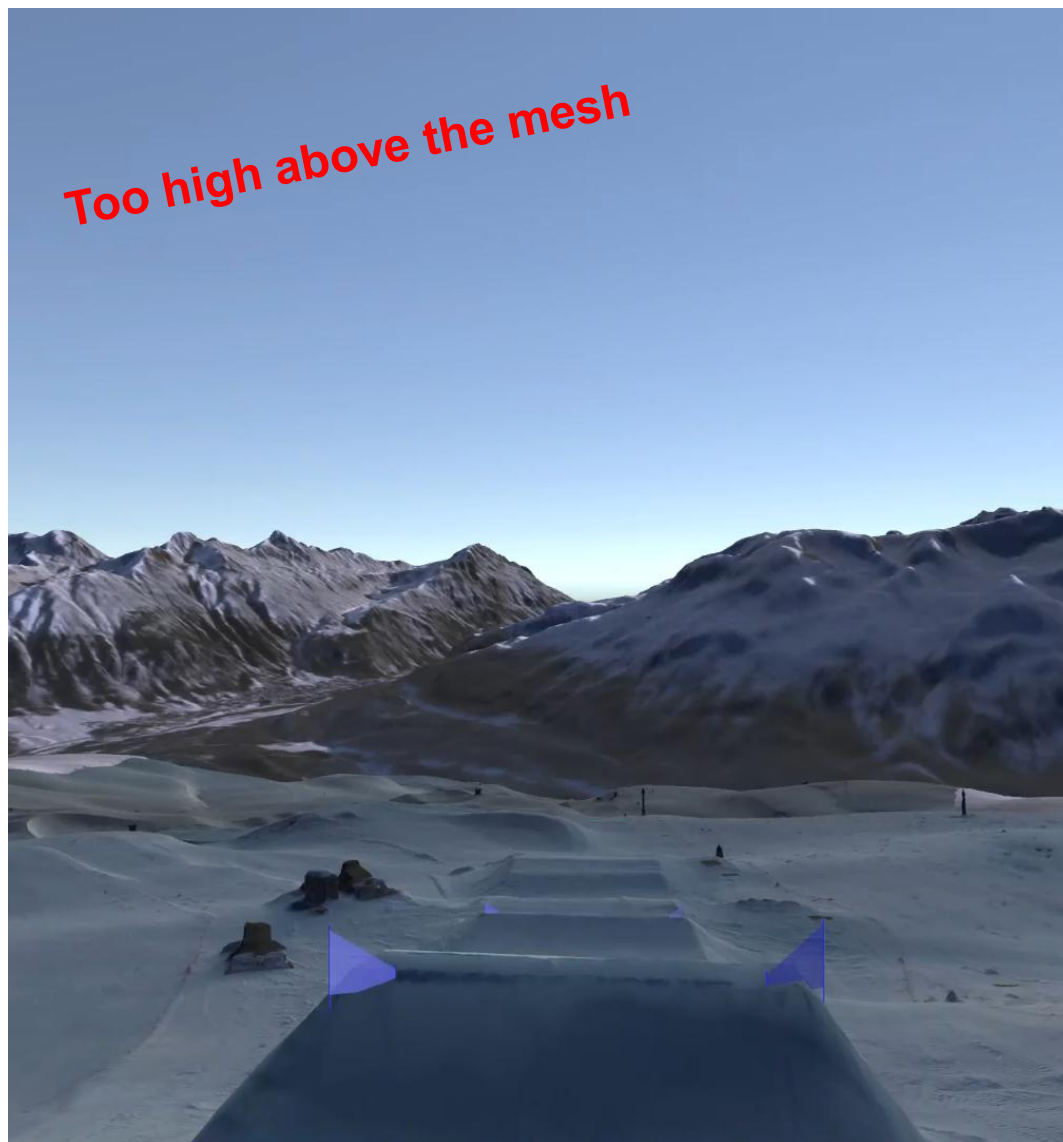
Trajectory (for example from GNSS)

Single Point Positioning (SPP) :  
Hz : ~2m V : ~6m

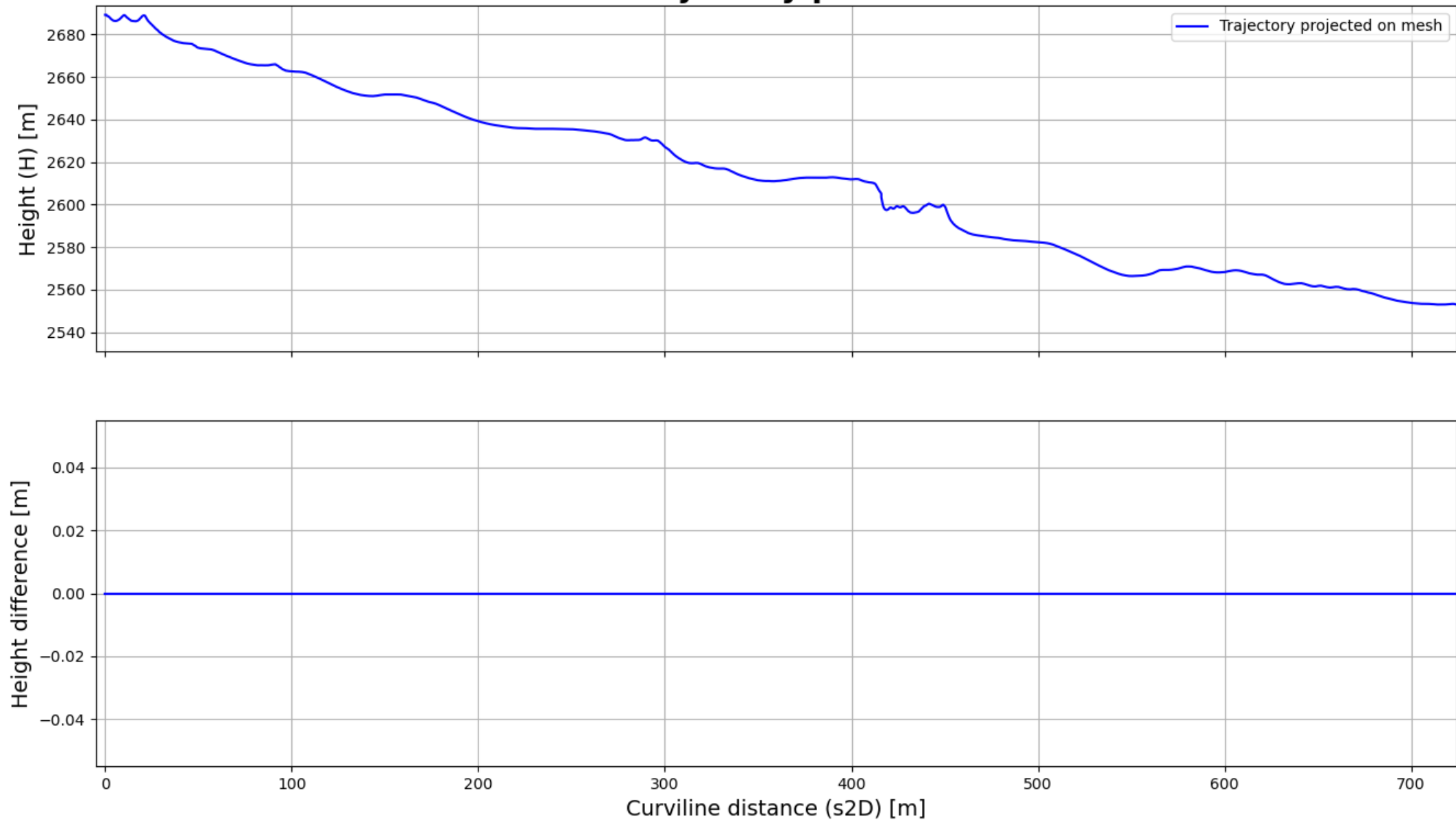
## Trajectory profile



# Motivation for filtering



## Trajectory profile

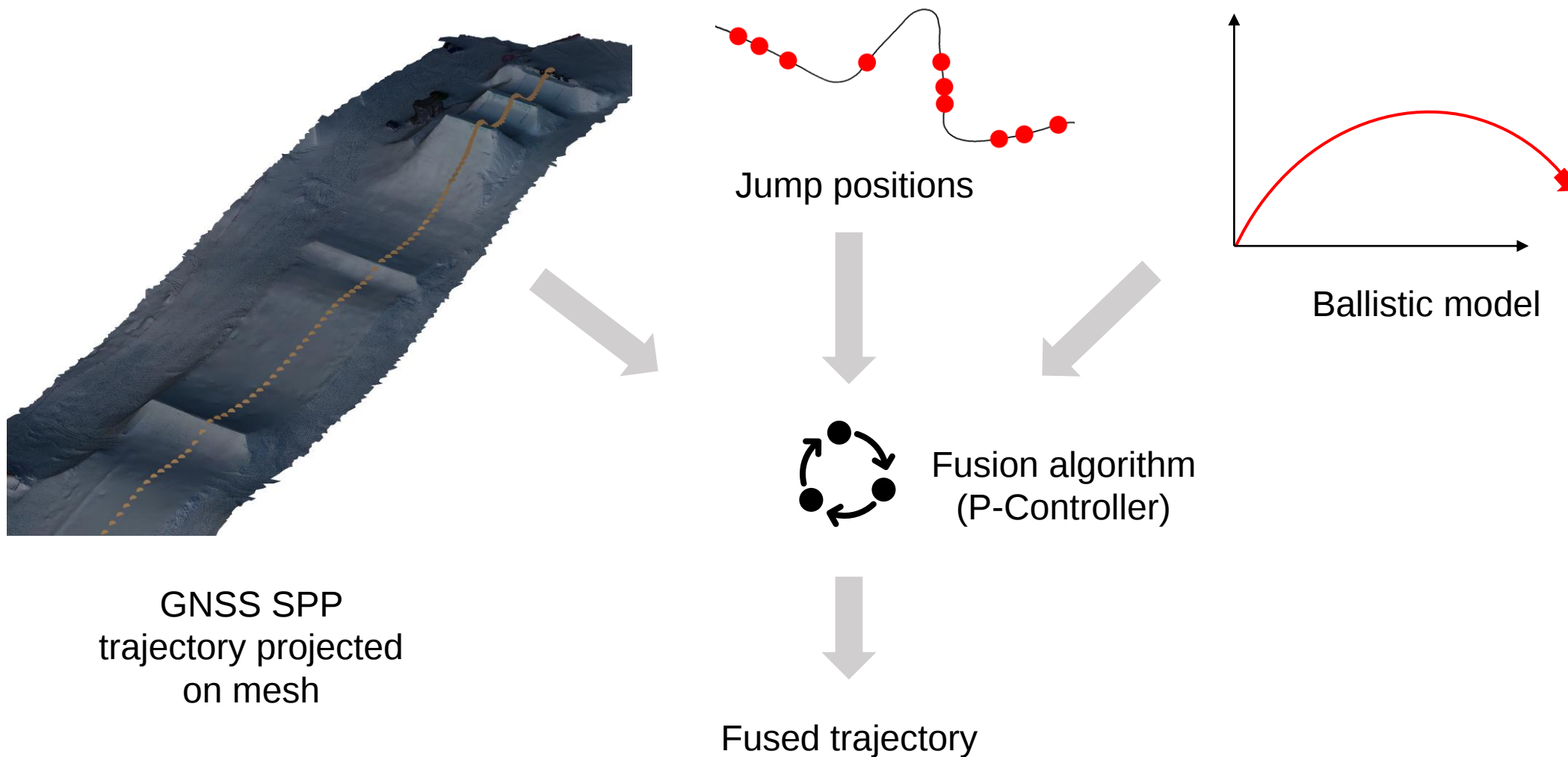


# Projection SPP trajectory on mesh

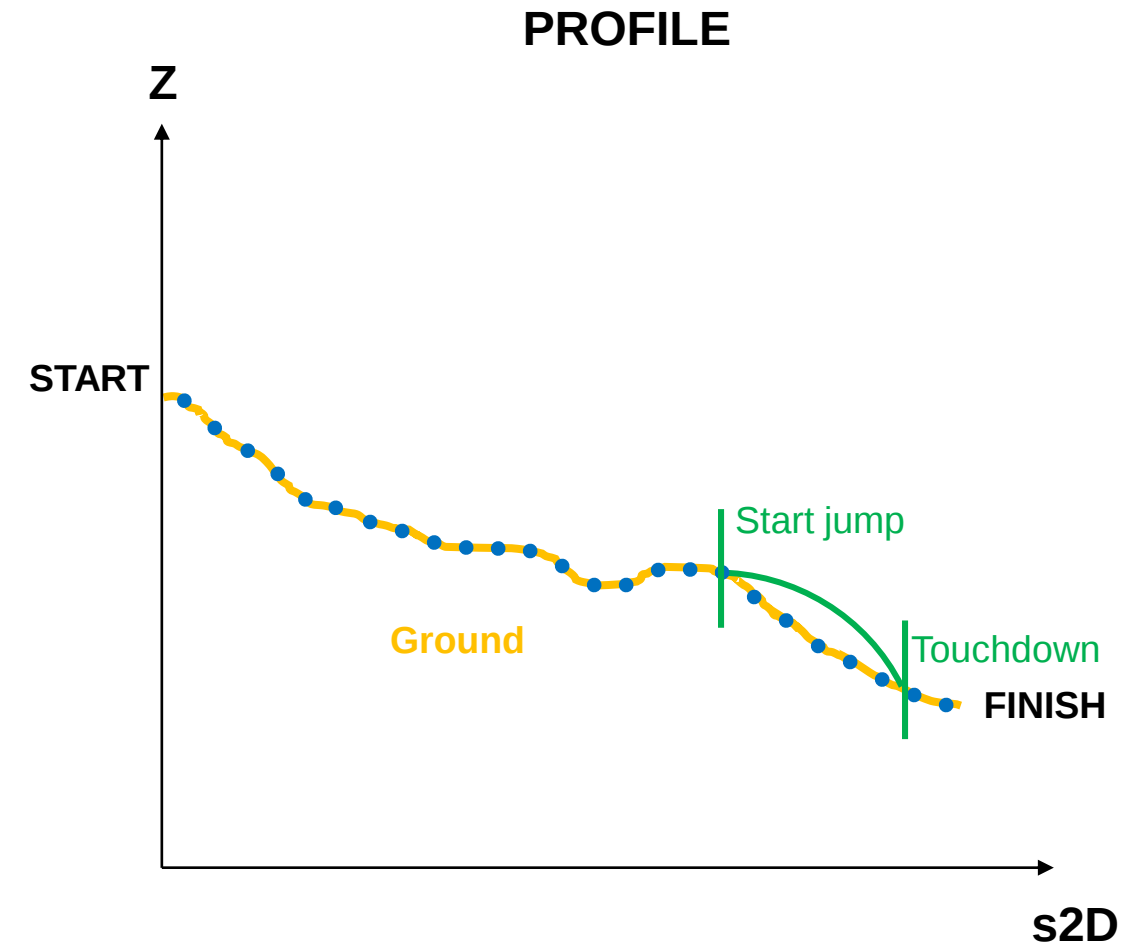
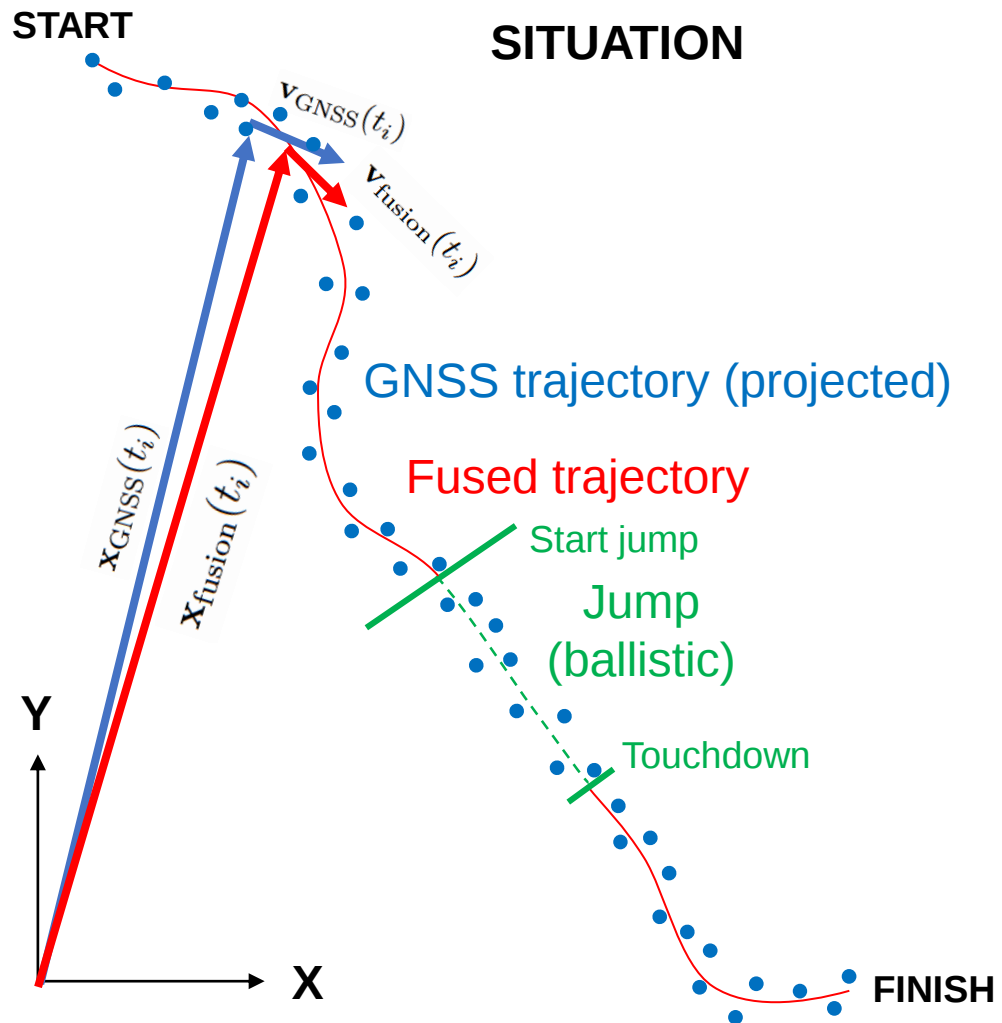
Better, but what about the jumps ?



# Fusion GNSS with ballistic model



# Fusion GNSS with ballistic model



# Fusion GNSS with ballistic model

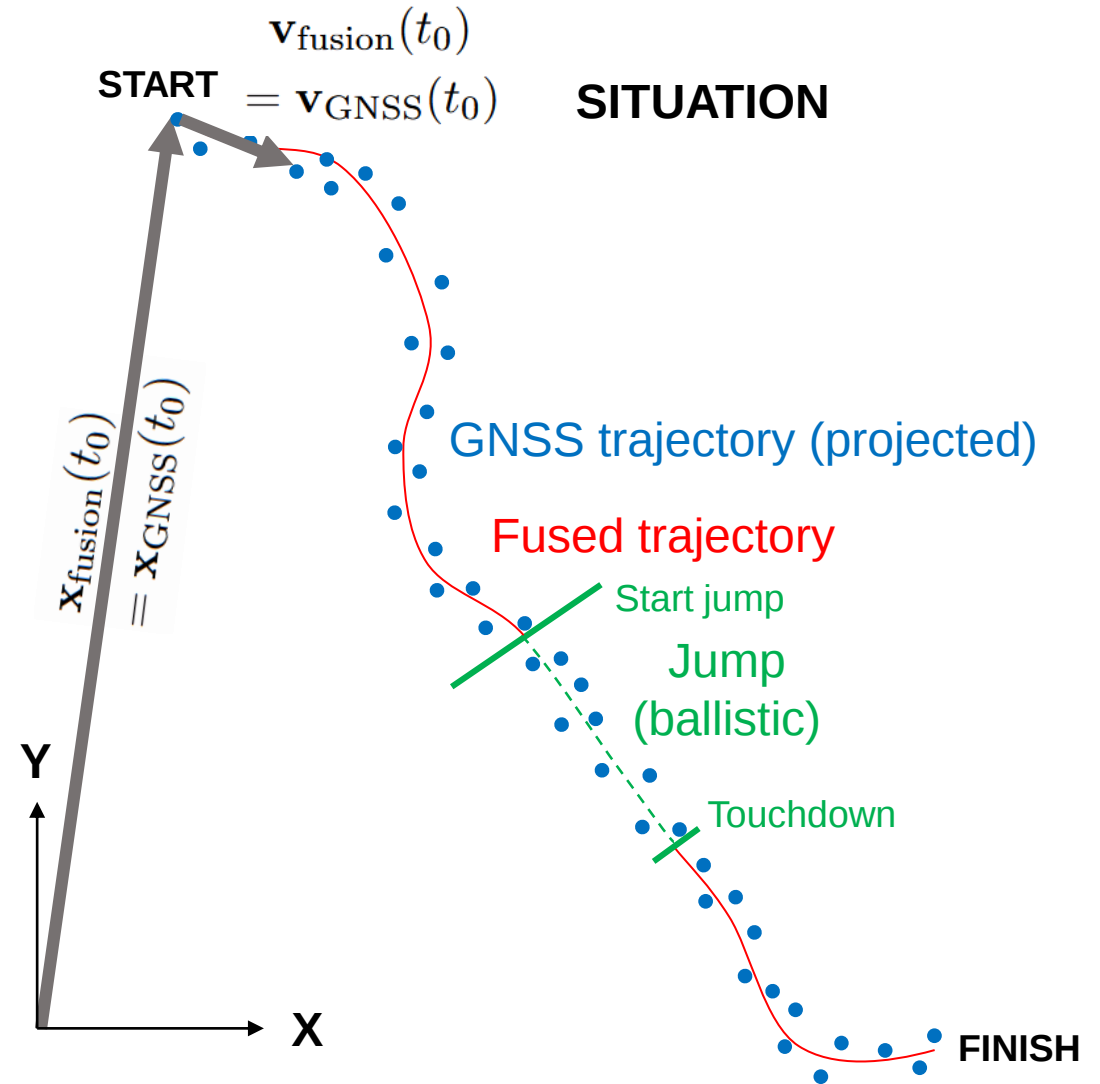
## Initial state vectors

$$\mathbf{x}_{\text{fusion}}(t_0) = \mathbf{x}_{\text{GNSS}}(t_0)$$

$$\mathbf{v}_{\text{fusion}}(t_0) = \mathbf{v}_{\text{GNSS}}(t_0)$$

$$\Delta t(t_0) = t_1 - t_0$$

$$\Delta \mathbf{v}(t_0) = \mathbf{0}$$



# Fusion GNSS with ballistic model

## Propagation of the state vectors on the ground

$$\mathbf{x}_{\text{fusion}}(t_{i+1}) = \mathbf{x}_{\text{fusion}}(t_i) + \mathbf{v}_{\text{fusion}}(t_i) \cdot \Delta t(t_i)$$

$$\mathbf{v}_{\text{fusion}}(t_{i+1}) = \mathbf{v}_{\text{fusion}}(t_i) + \Delta \mathbf{v}(t_i)$$

With :

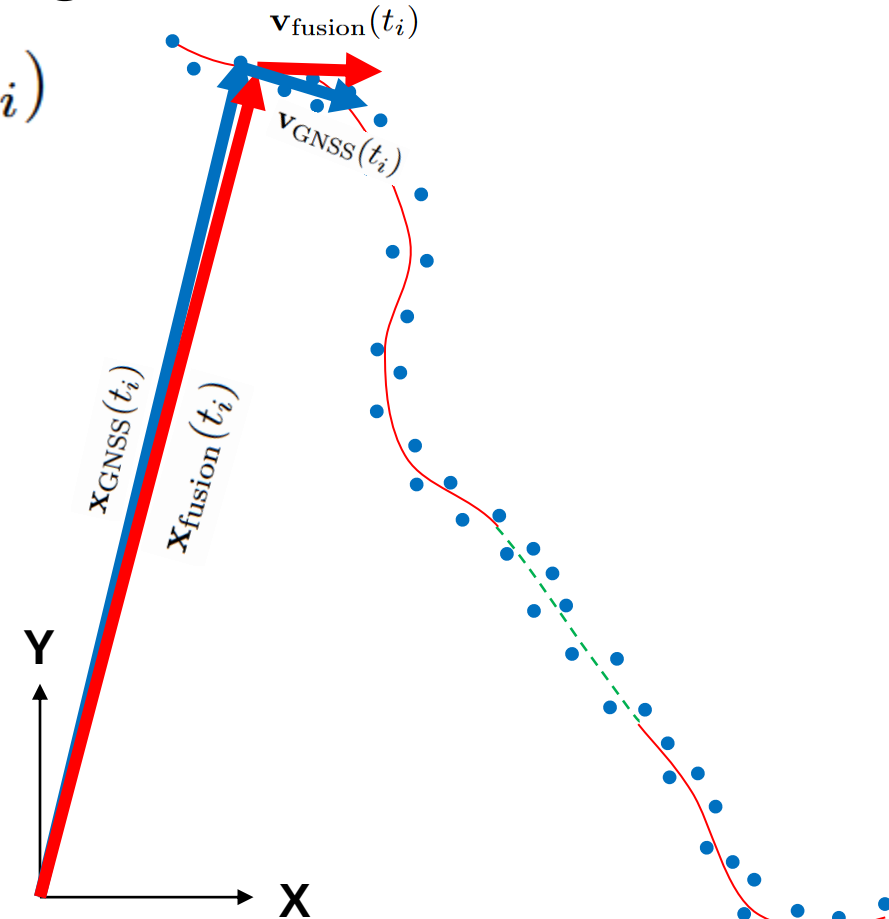
$$\Delta t(t_i) = t_{i+1} - t_i$$

simple P-Controller  
for **position** and **velocity** matching

$$\Delta \mathbf{v}(t_i) = -k_{\mathbf{x}} \cdot \mathbf{e}_{\mathbf{x}}(t_i) - k_{\mathbf{v}} \cdot \mathbf{e}_{\mathbf{v}}(t_i)$$

$$\mathbf{e}_{\mathbf{x}}(t_i) = \mathbf{x}_{\text{GNSS}}(t_i) - \mathbf{x}_{\text{fusion}}(t_i)$$

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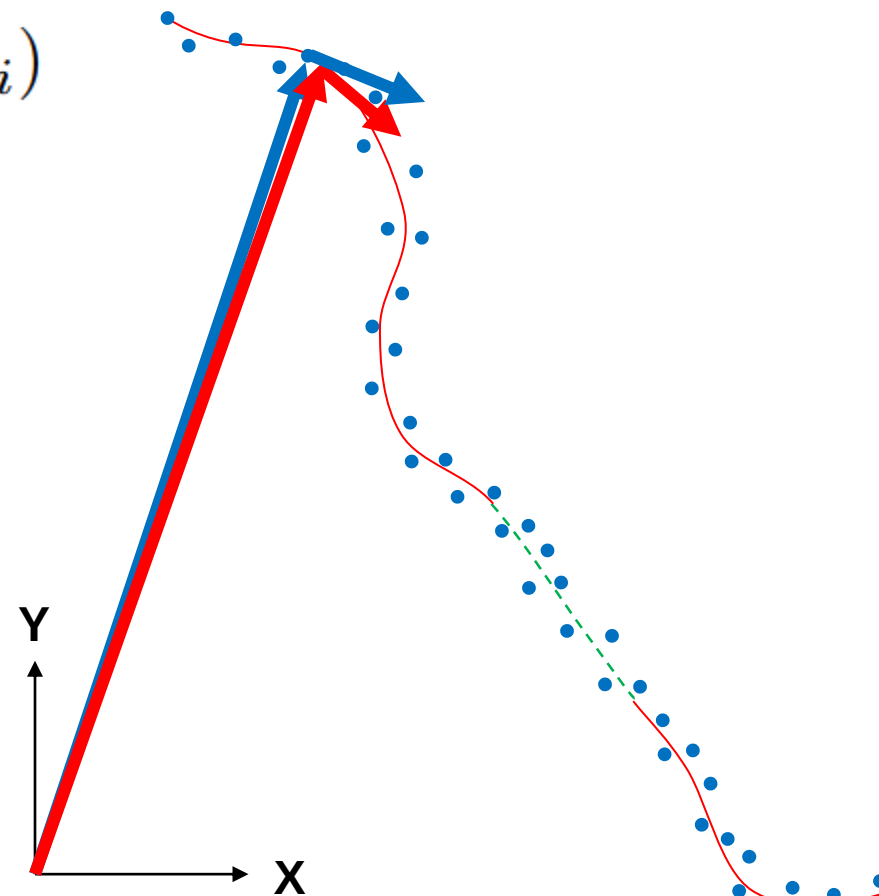
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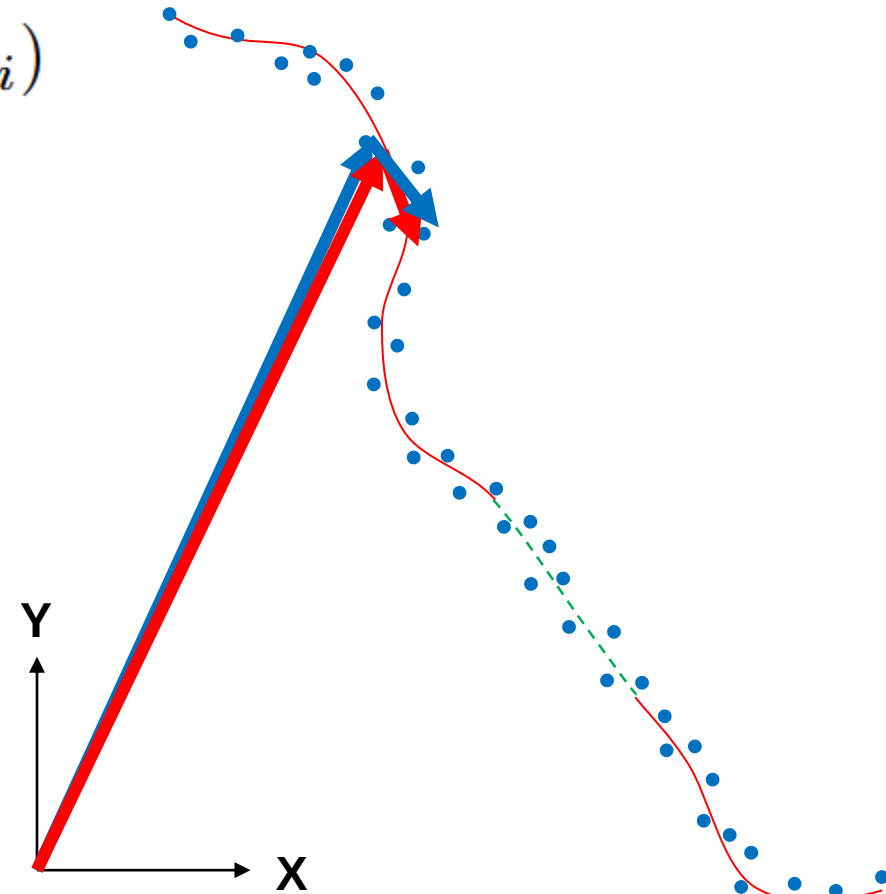
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With :

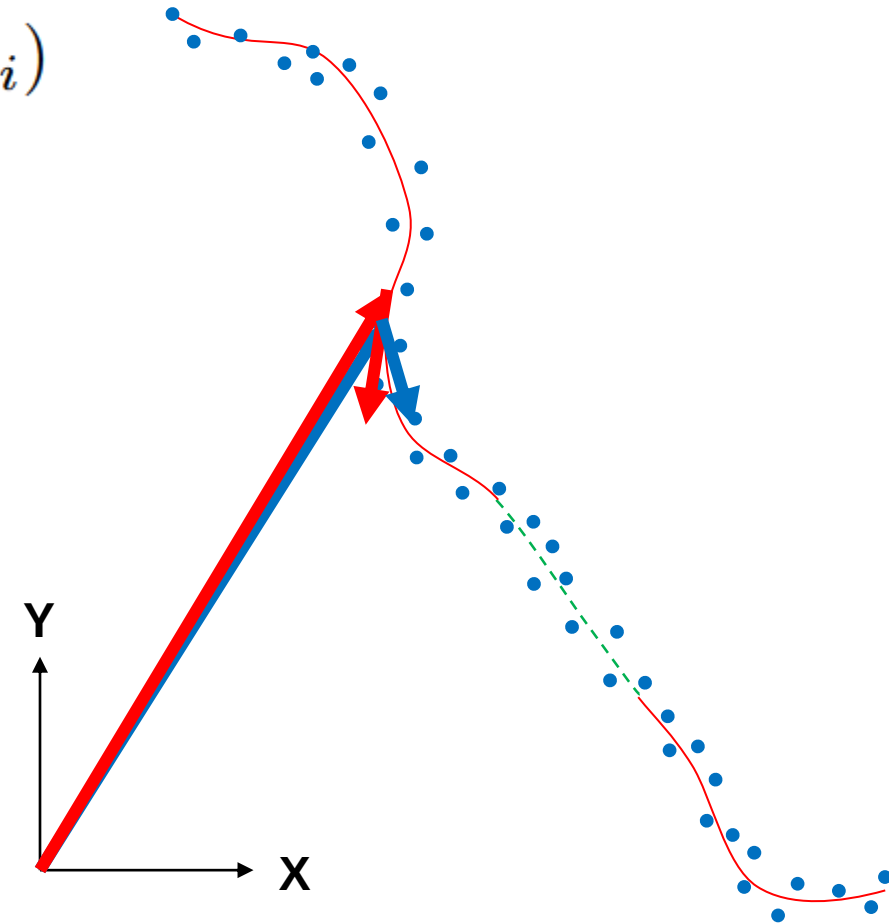
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With :

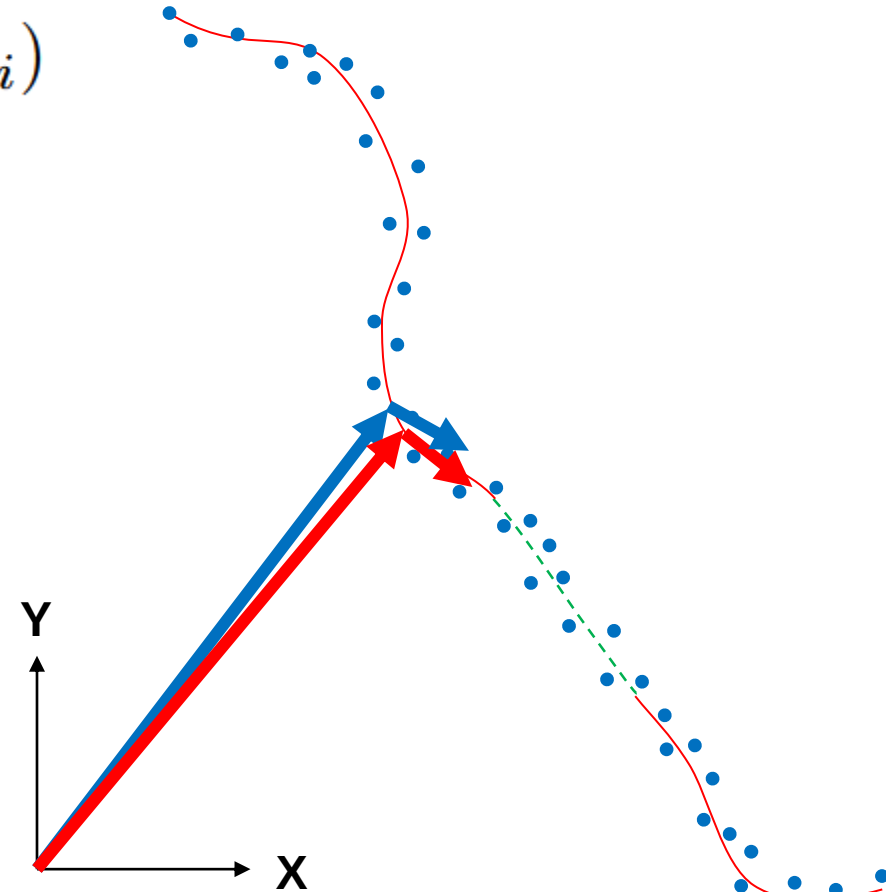
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# Fusion GNSS with ballistic model

## Initial conditions for ballistic integration (jump start)

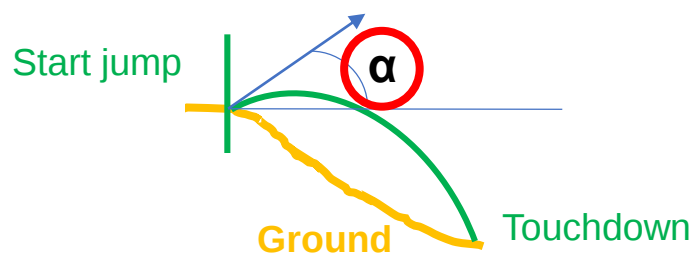
$$\mathbf{x}_{\text{fusion}}(t_{\text{jump}_0^j}) = \mathbf{x}_{\text{fusion}}(t_i)$$

$$\mathbf{v}_{\text{fusion}}(t_{\text{jump}_0^j}) = \mathbf{v}_{\text{fusion}}(t_i) + \Delta \mathbf{v}_{\text{impulse}^j}$$

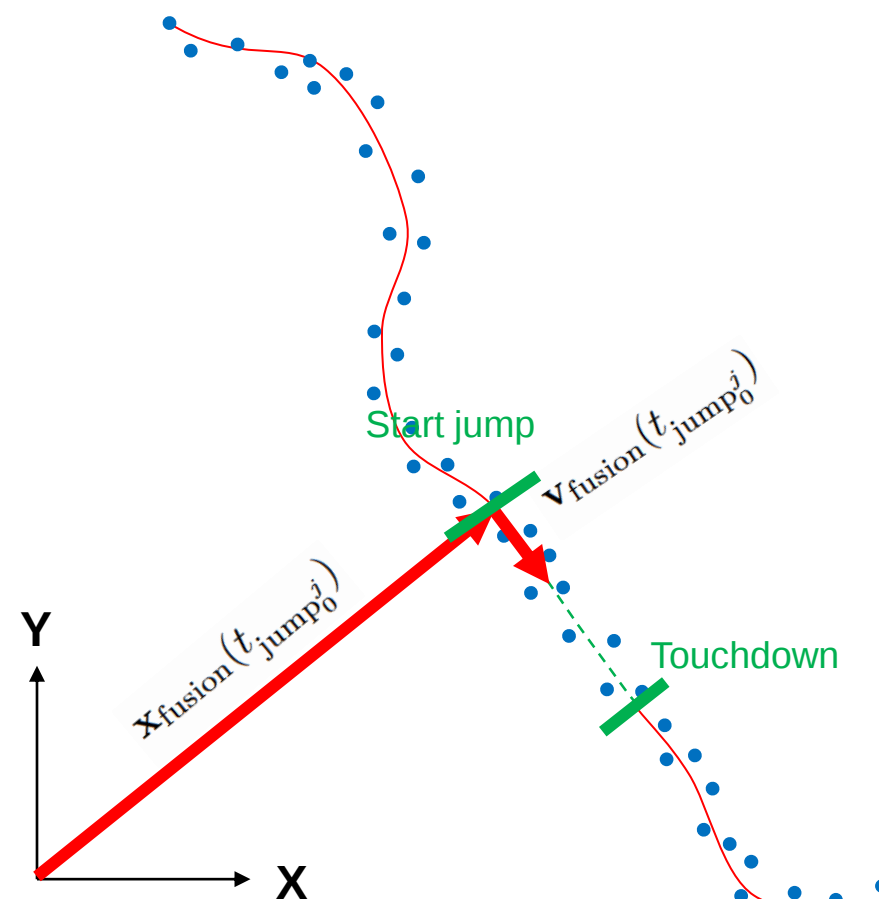
With :

$$\Delta \mathbf{v}_{\text{impulse}^j} = \begin{pmatrix} 0 \\ 0 \\ \alpha[\text{rad}] \cdot |\mathbf{v}_{\text{fusion}}(t_i)| \end{pmatrix}$$

### PROFILE



### SITUATION



# Fusion GNSS with ballistic model

## Propagation of the state vectors in flight

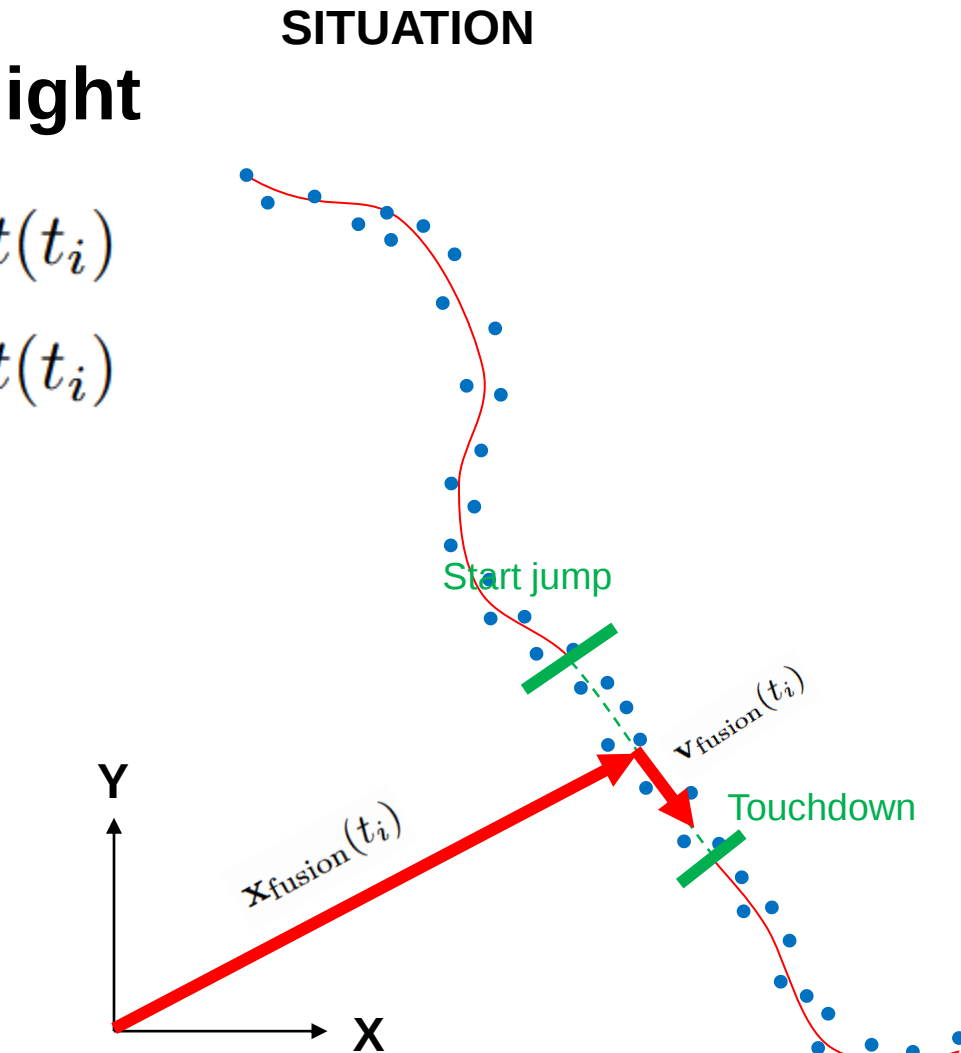
$$\mathbf{x}_{\text{fusion}}(t_{i+1}) = \mathbf{x}_{\text{fusion}}(t_i) + \mathbf{v}_{\text{fusion}}(t_i) \cdot \Delta t(t_i)$$

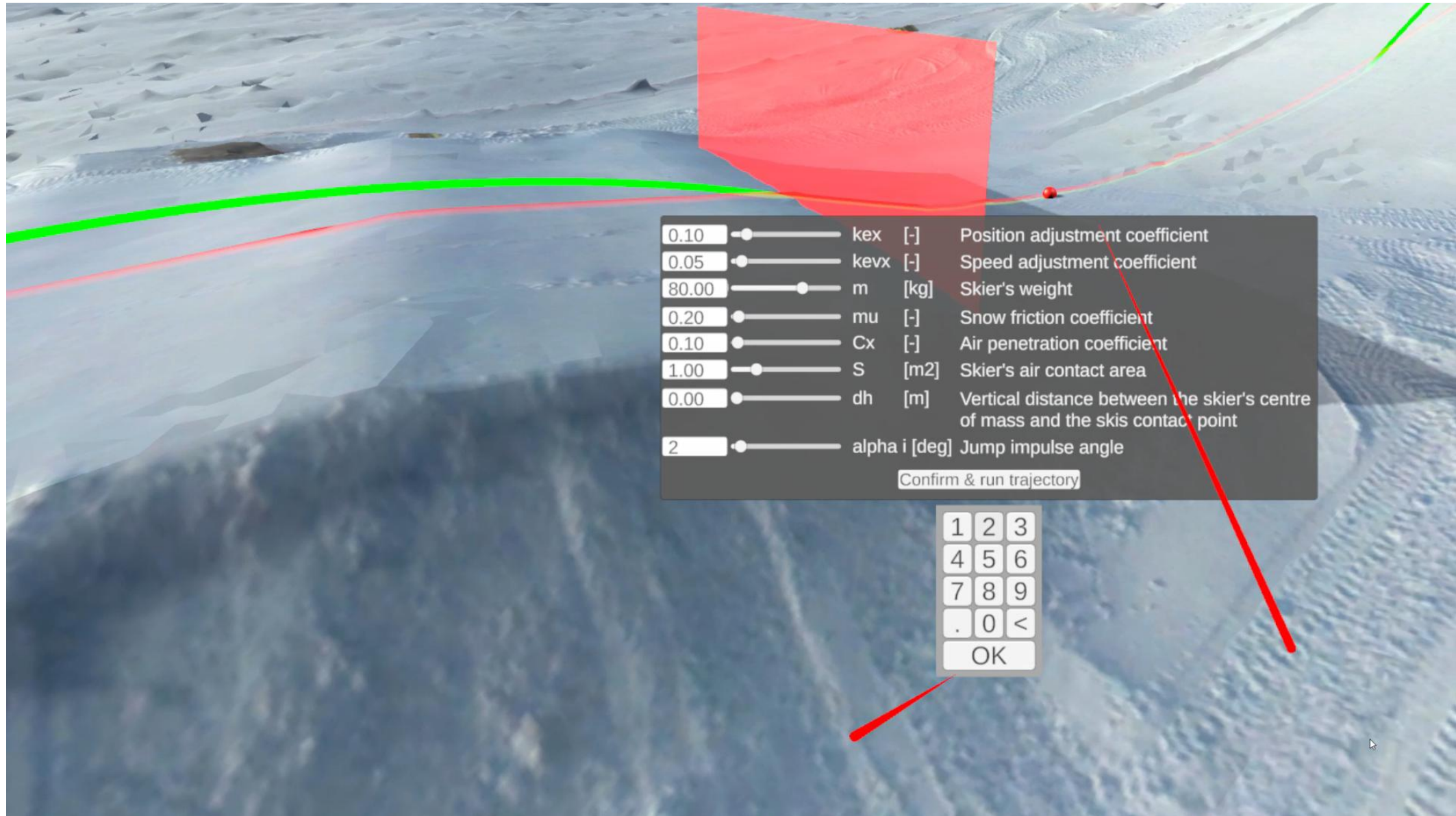
$$\mathbf{v}_{\text{fusion}}(t_{i+1}) = \mathbf{v}_{\text{fusion}}(t_i) + \mathbf{a}_{\text{fusion}}(t_i) \cdot \Delta t(t_i)$$

With :

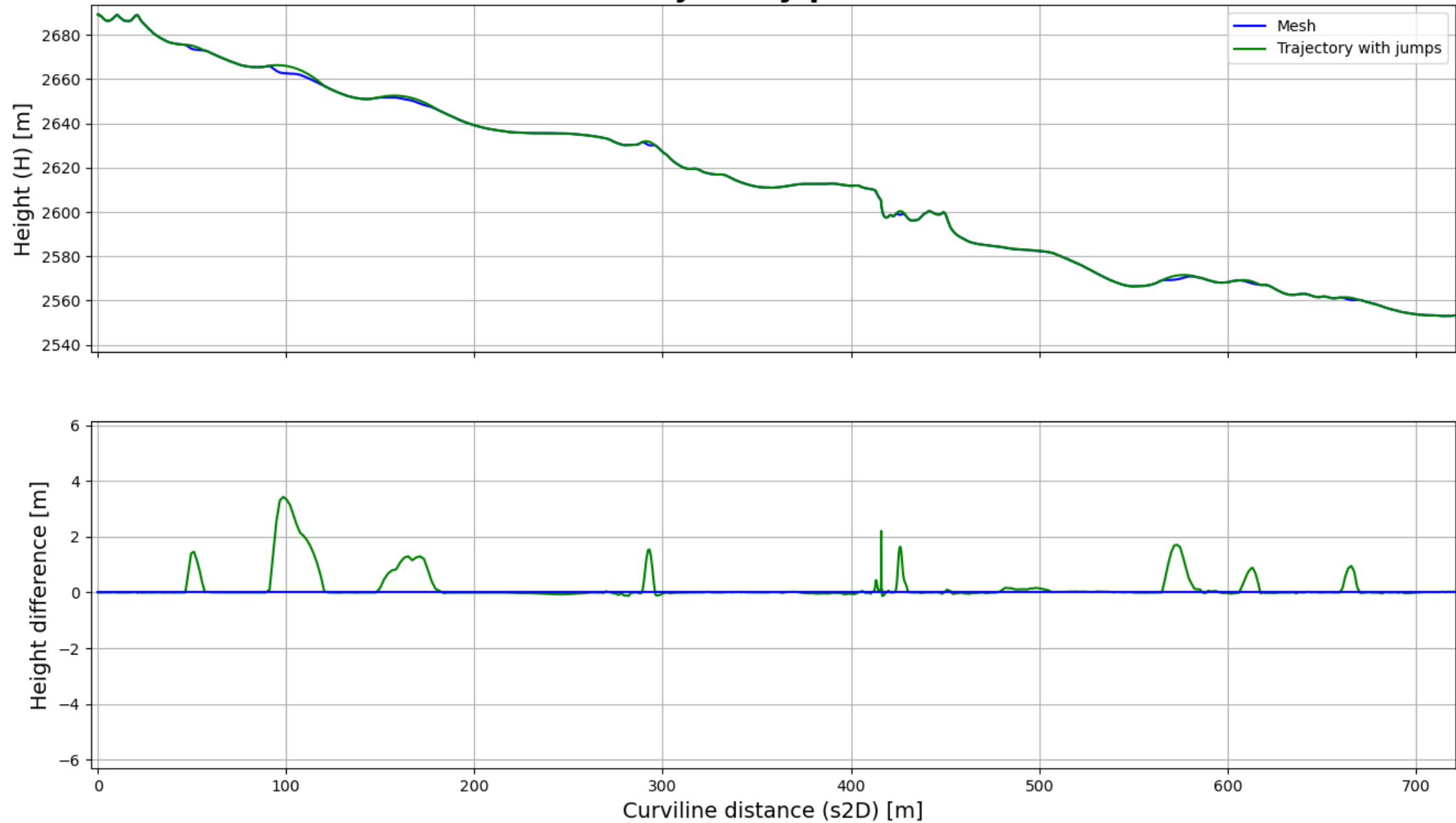
$$\mathbf{a}_{\text{fusion}}(t_i) = \mathbf{g} + \frac{1}{m} \cdot \mathbf{F}_{\text{airdrag}}$$

$$\mathbf{F}_{\text{airdrag}} = -\frac{1}{2} \cdot \rho_{\text{air}} \cdot C_x \cdot S \cdot |\mathbf{v}_{\text{fusion}}(t_i)|^2 \cdot \frac{\mathbf{v}_{\text{fusion}}(t_i)}{|\mathbf{v}_{\text{fusion}}(t_i)|}$$

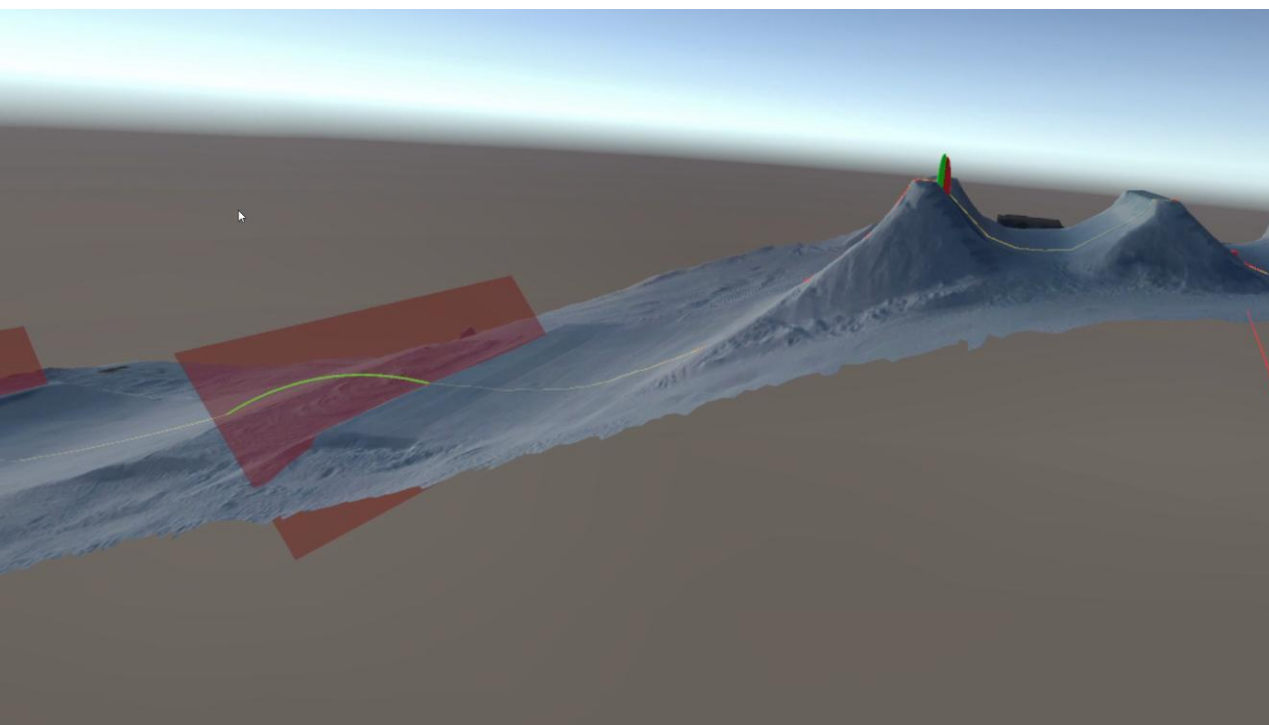




## Trajectory profile



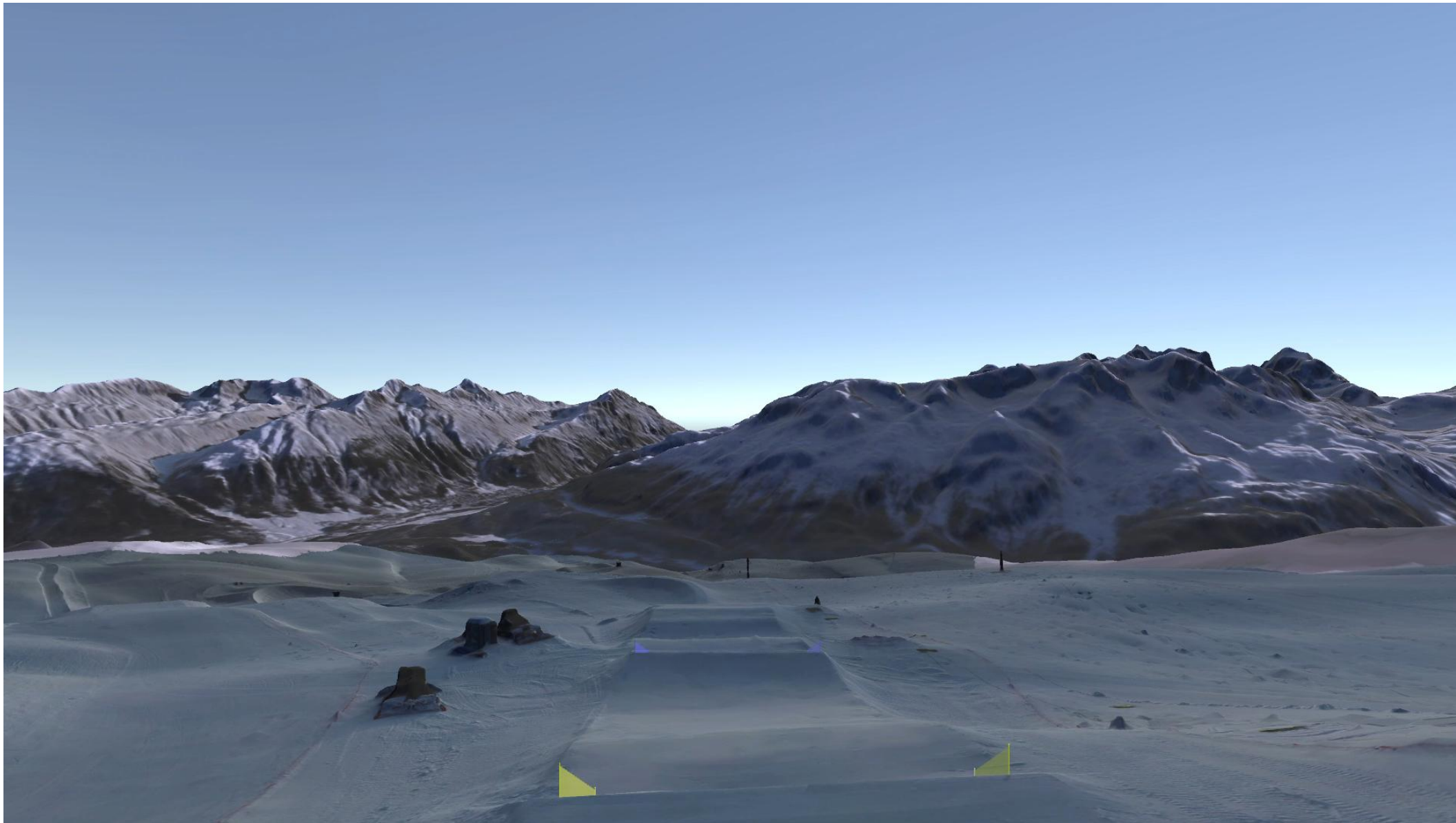
# Visual check with video comparison



Data : HEIG-VD

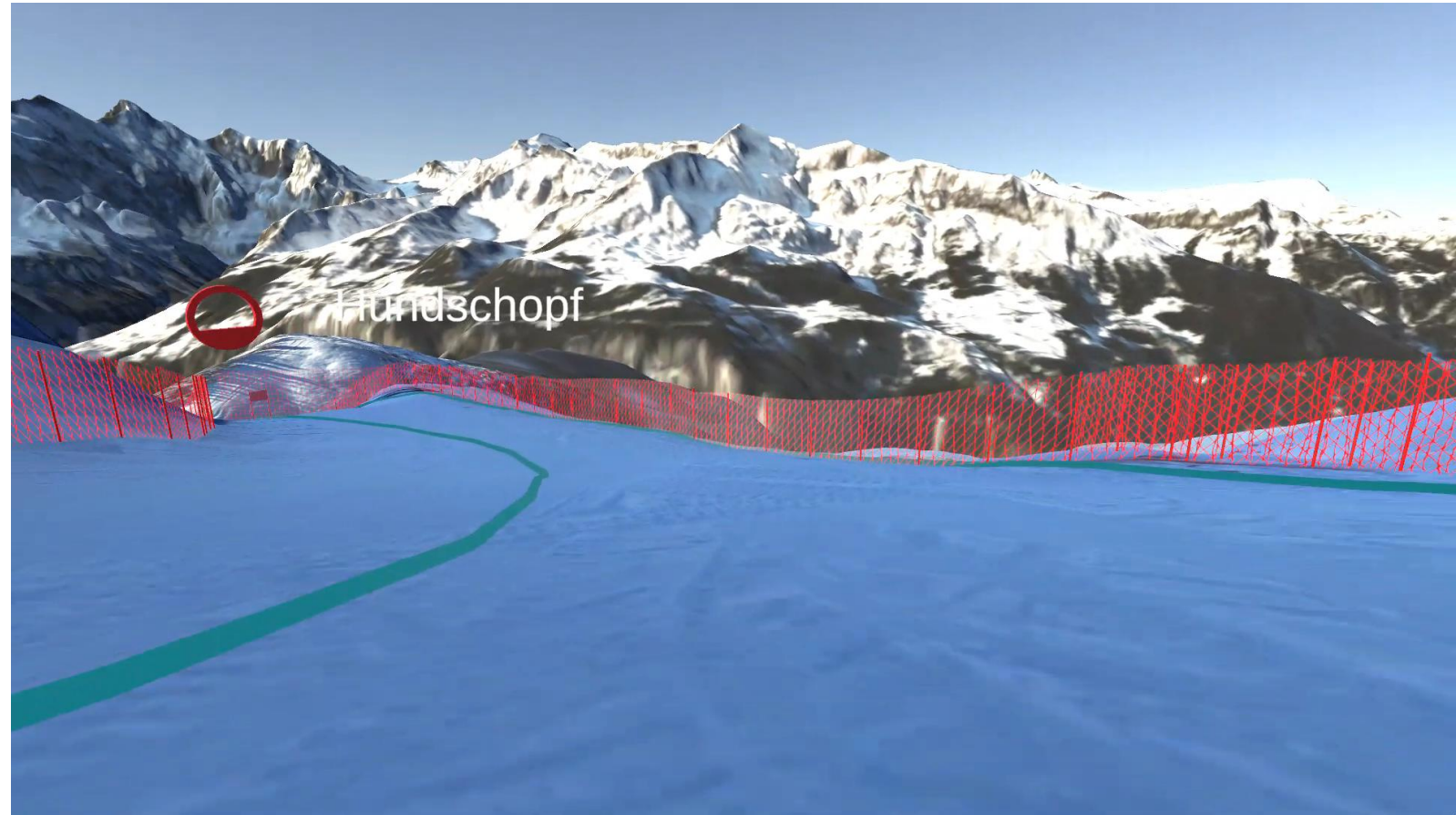


Video : Swiss-Ski – Björn Bruhin



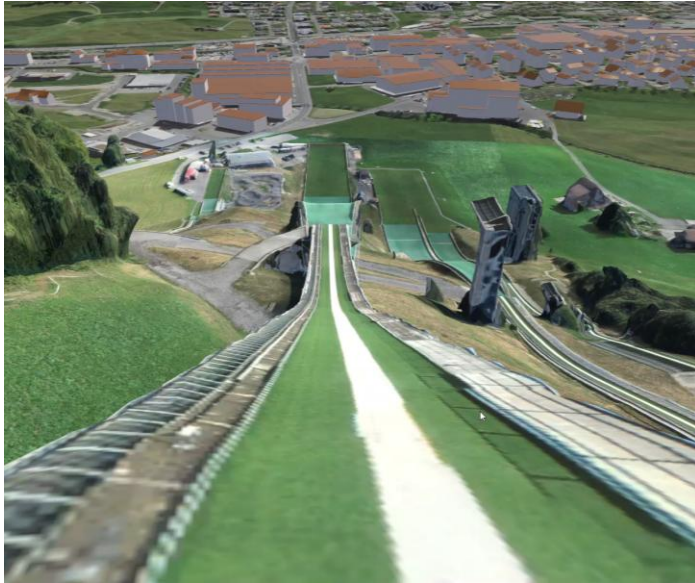
# Conclusion

- Useful fusion-algorithm :
  - When trajectory with bad vertical component (GNSS SPP)
  - When no measured trajectory available
- Further usecases :
  - Alpine skiing
  - Ski jumping
  - ...



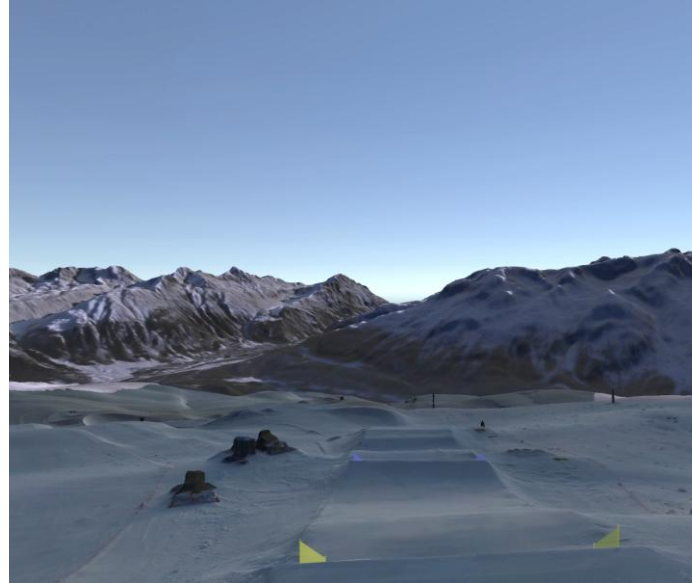
# Thank you for your attention!

Einsiedeln



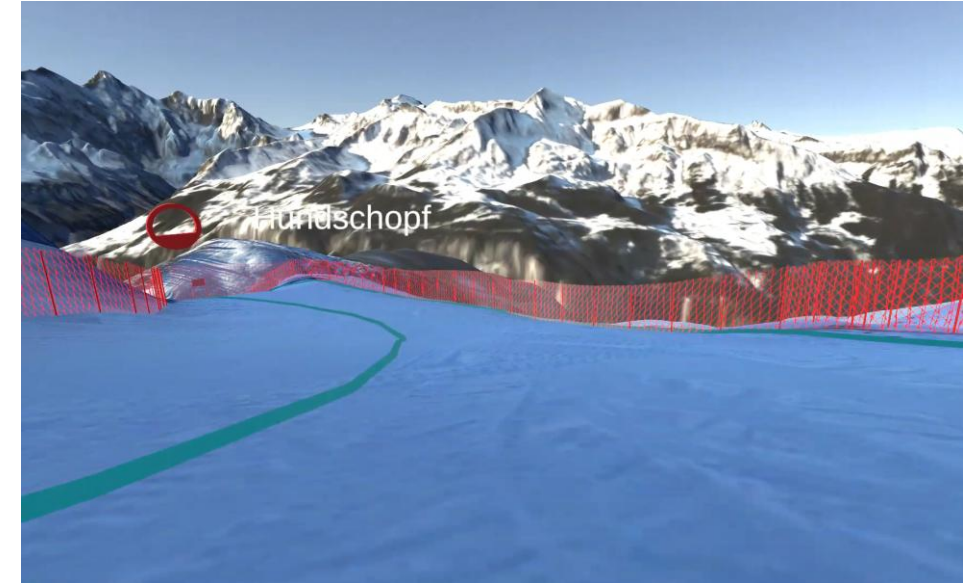
Data : HEIG-VD, swisstopo

St. Moritz



Data : HEIG-VD, swisstopo, Sentinel

Wengen



Data : HEIG-VD, swisstopo, Sentinel

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